Circuit Model of Circuit of Willis to Predict Stroke

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Background

- Circle of Willis (CoW) is a network of arteries in brain stem that supplies oxygenated blood to >80% of the brain. Primary location where most strokes occur.
- Hypothesis: Circuit model of Circle of Willis identifies arterial regions prone to high stroke risk.
- Goal: Predict individuals who are more prone to stroke

Methodology

Modeling blood pressure difference using circuit analysis to calculate blood flow rate

- Potential Difference: ΔV
- Electrical Resistance: r
- Electrical Current: i
- Pressure Difference: ΔP
- Dynamic Viscosity: µ
- $\text{Length}_{\text{Artery}}$: L
- $\text{Radius}_{\text{Artery}}$: R
- Volumetric Flow Rate: Q

•Poiseuille's Law: $\Delta P = \frac{8\mu L}{\pi R4} * Q$

•Ohm's Law: $\Delta V = r * i$

•Five sets of clinical data on artery dimensions used to compute models

•22.5 billion (= 4.5 billion * 5 sets of anatomical data) simulated

•Flow rates through inlets, i.e. ICA, BA also chosen from ranges provided in literature

•Utilized MATLAB (Parallel Computing & Symbolic Math Toolbox) & HPC (High Performance Clusters) at RCDC, University of Houston

•Models validated with 3 sets of empirical clinical data on volumetric flow rates

•Chose successful models defined as:

$$\text{flow rate}^{\text{Model}} \ \varepsilon \ [\text{flow rate}^{\text{Clinical}}_{\text{min}}, \text{flow rate}^{\text{Clinical}}_{\text{max}}]$$

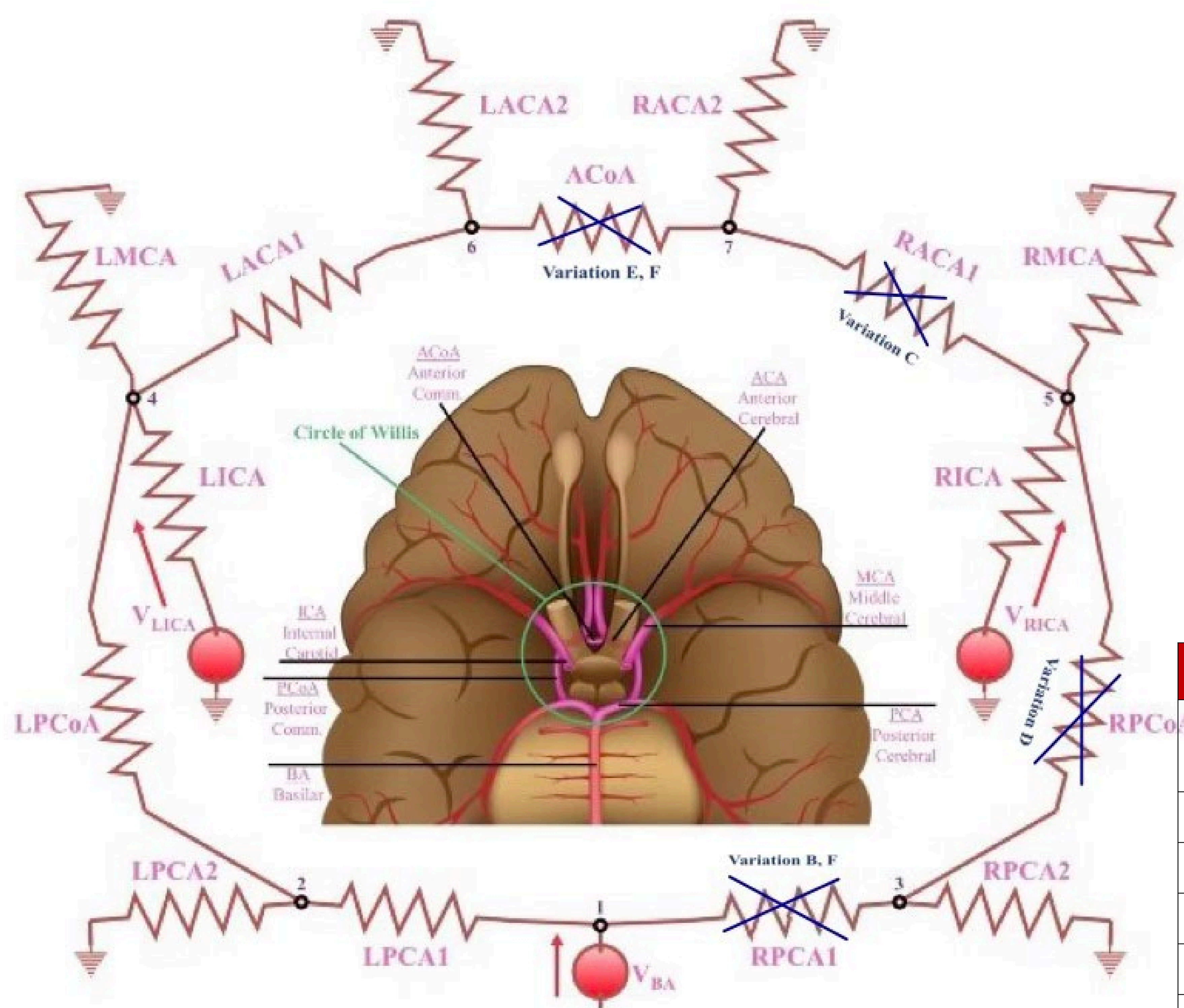

Figure 1. Anatomical and Circuit Representation of the CoW

(Var A:Complete CoW [~51% of pop], Var B:Missing PCA1 [~18% of pop], Var C:Missing ACA1 [~5% of pop], Var D,E,F [~6 %])

Circuit Equations:

1) $V_{BA}=V_1$

2) $\frac{V_2-V_1}{R_{LPCA1}} + \frac{V_2}{R_{LPCA2}} + \frac{V_2-V_4}{R_{LPCoA}} = 0$

3) $\frac{V_3-V_1}{R_{RPCA1}} + \frac{V_3}{R_{RPCA2}} + \frac{V_3-V_5}{R_{RPCoA}} = 0$

4) $\frac{V_4-V_2}{R_{LPCoA}} + \frac{V_4-V_{LICA}}{R_{LICA}} + \frac{V_4}{R_{LMCA}} + \frac{V_4-V_6}{R_{LACA1}} = 0$

5) $\frac{V_5-V_3}{R_{RPCoA}} + \frac{V_5-V_{RICA}}{R_{RICA}} + \frac{V_5}{R_{RMCA}} + \frac{V_5-V_7}{R_{RACA1}} = 0$

6) $\frac{V_6-V_4}{R_{LACA1}} + \frac{V_6-V_7}{R_{ACoA}} + \frac{V_6}{R_{LACA2}} = 0$

7) $\frac{V_7-V_5}{R_{RACA1}} + \frac{V_7-V_6}{R_{ACoA}} + \frac{V_7}{R_{RACA2}} = 0$

Name	Artery	Variation
Var. A	Contains all arteries	Complete
Var. B	PCA	Hypoplasia
Var. C	ACA	Hypoplasia
Var. D	PCoA	Hypoplasia
Var. E	ACoA	Hypoplasia
Var. F	PCA and ACoA	Double Hypoplasia

RESULTS

Table 1. Number of Models with 0 Error from each dataset

Variations	Iqbal[3]	Hillen[4]	Maaly[6]	Shatri[2]	Karatas[5]
A	35,873,206	28,072,005	81,876,890	72,084,374	52,395,340
A&B	1,039,592	5,887,634	227,013	625,167	269,011
A&C	4,261,205	3,232,147	2,014,354	5,295,803	16,868,450
A&B&C	66,746	36,453	1,749	15,387	47,071

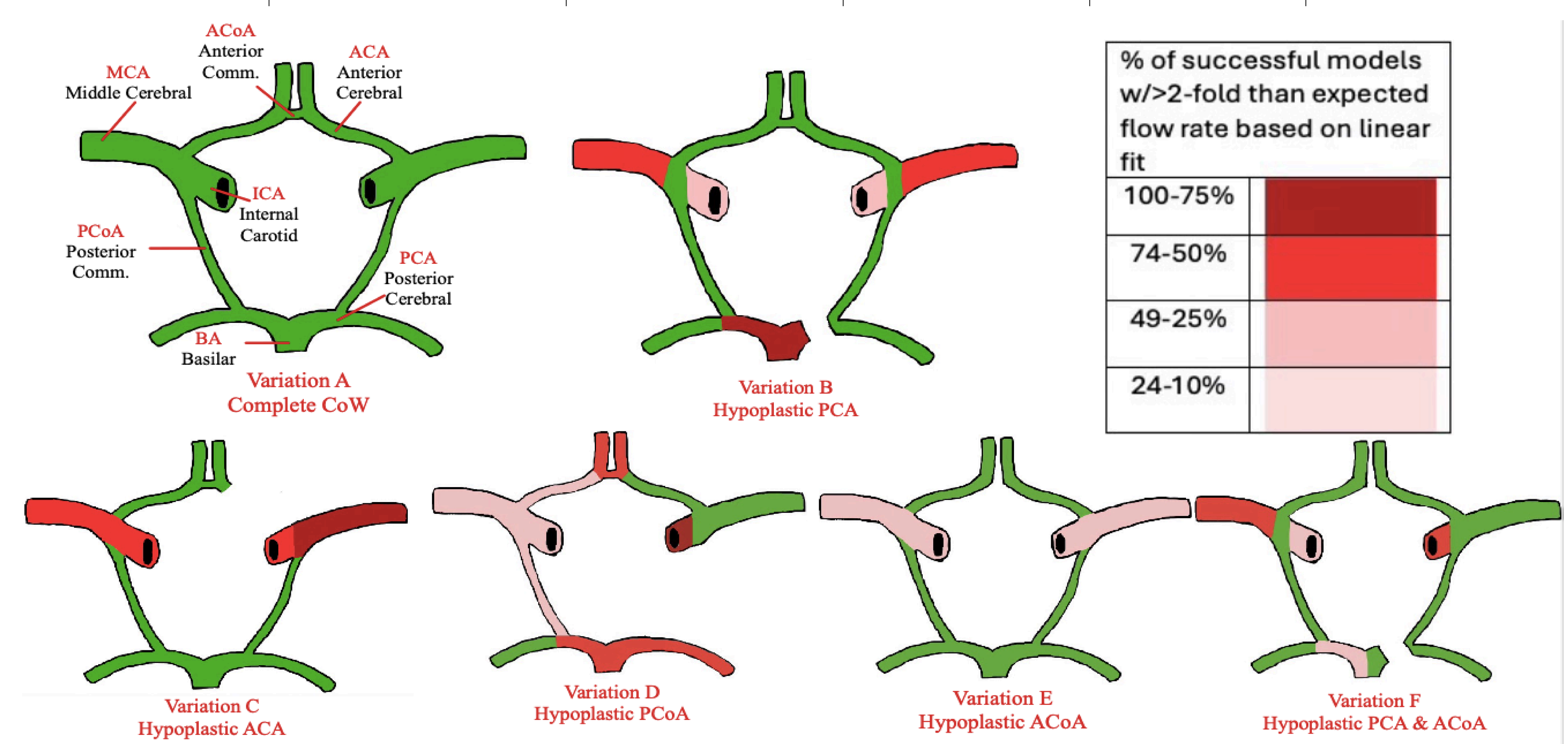

Figure 2. CoW Variations experiencing higher than expected flow rate

SUMMARY

- **165,000+ circuit models obey anatomical measures and yield empirically verified flow rates within the CoW.**
- Generated 22.5B simulations, with 265M successful models for Var. A, representing ~50% of population.
- MCA identified as most turbulent with highest flow rates; 50.8% of strokes occur in MCA region
- Simulated unexplored Vars. D, E, F using validated models from Vars. A,B and C.

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References

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